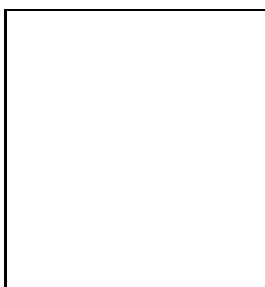


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# GRAVITATIONAL STABILITY AND SCREENING EFFECT FROM EXTRA TIMELIKE DIMENSIONS <sup>a</sup>

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We discuss extra timelike dimensions and their effects on the gravitational stability of spherical massive bodies. Here we specifically report our results for the case of one extra timelike dimension where we have made analytically rigorous investigations on the tachyonic graviton exchange due to the infinite tower of the Kaluza-Klein mode. With the scale  $L$  of the extra timelike dimension we find that some spherical bodies of radius  $R$  can be stable at critical radii  $R = 2\pi Lp$  for some positive integer  $p$ . We also obtain the generic property of massive bodies that for the short distance range  $0 < R \leq \pi L$  the gravitational force due to the ordinary massless graviton exchange is screened by the Kaluza-Klein mode exchange of tachyonic gravitons.

## 1 Physics of Extra Timelike Dimensions

### 1.1 Possibility for extra times

There is no a priori reason why extra times cannot exist. A possibility of the existence of  $D$  extra timelike dimensions is an interesting subject in its own right and has also been discussed in various contexts in the past: the subject has been studied in cosmological constant problem by Aref'eva *et al* <sup>1</sup>, in string theories by Vafa <sup>2</sup> *et al*, in brane theories by Chaichian *et al* <sup>3</sup>, in supergravity by Popov <sup>4</sup> *et al*, and in many other topics such as the ones by Sakharov <sup>5</sup>, Chamblin *et al* <sup>6</sup>.

### 1.2 Experimental Viability and Signatures

Phenomenology and observable physical effects of extra timelike dimensions have been discussed by Ynduráin <sup>7</sup> and by Dvali *et al* <sup>8</sup> in a scenario where the Standard Model particles are localized

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in “our time”, whereas gravity can propagate in all time directions.

The compactification of extra timelike space gives rise to the Kaluza-Klein mode of tachyons and causes the violation of causality and conserved probability. The gravitational potential turns out to have an imaginary part which makes massive bodies unstable. But the observable effect is not unacceptable experimentally if the scale  $L$  of the extra dimensions is bounded below a sufficiently small size<sup>7</sup> which is nearly as close as to the Planck scale.

One may naturally ask what then are other noticeable signatures of extra timelike dimensions. In this report we aim at definite rigorous study of this question being based on an analytic ground.

## 2 Gravitational Stability and Screening Effect

### 2.1 Gravitational Potential

We consider one extra timelike dimension with  $D = 1$  and compactify it on a circle of radius  $L$ . We shall look into the problem of the Kaluza-Klein tachyonic mode in detail based on an analytically precise treatment, thus obtaining the rigorous answer to the problem of instability of massive bodies<sup>9</sup>.

Let gravitons propagate in the extra dimension; then we obtain tachyonic gravitons of the Kaluza-Klein mode. Their propagators are:

$$-i \frac{1}{k_0^2 - \mathbf{k}^2 + \frac{n^2}{L^2} + i\epsilon}, \quad n = \text{integers} \quad (1)$$

up to a spin tensor factor. Then the gravitational potential between two unit mass points at distance  $d$  is given by

$$V(d) = -G_N \frac{1}{d} - \sum_{n=-\infty, n \neq 1}^{+\infty} G_N \frac{1}{d} e^{i \frac{|n|}{L} d} \sim \frac{1}{d^{1+1}} \quad \text{as } d \rightarrow 0 \quad (2)$$

in the nonrelativistic tree-level approximation, where  $G_N$  is the Newton constant.

We point out here that the complex gravitational potential used by Ynduráin<sup>7</sup> and also by Dvali *et al*<sup>8</sup> has a wrong sign in the phase factor, and that the correct sign of the phase together with the correct overall sign of the potential is crucially important in discussing the stability of matter from the aspect of its *vanishing* or *explosion*.

### 2.2 Gravitational Self-Energy

The gravitational self-energy of a spherical body of radius  $R$  with mass density  $\rho(r)$  is calculated as

$$E_n(R) = 8i\pi^2 G_N L \int_0^R dr \int_0^r d\ell \rho(r) \rho(\ell) \frac{r\ell}{|n|} \left[ e^{i \frac{|n|}{L}(r+\ell)} - e^{i \frac{|n|}{L}(r-\ell)} \right], \quad (3)$$

which gives for  $n \rightarrow 0$

$$E_0 = -16\pi^2 G_N \int_0^R dr \int_0^r d\ell \rho(r) \rho(\ell) r\ell^2. \quad (4)$$

The total gravitational self-energy is then:

$$E(R) = E_0 + \sum_{n=-\infty, n \neq 0}^{+\infty} E_n = E_0 + 2 \sum_{n=1}^{\infty} E_n. \quad (5)$$

### 2.3 Gravitational Stability and Screening Effect

Let us choose the mass density  $\rho(r) = D/r$ . Then we get the imaginary part of the self-energy as

$$\Im E(R = 2\pi Lk + c) = -16\pi^2 G_N D^2 L \int_0^c dr \int_0^r d\ell \ln \left| \frac{\sin \frac{r+\ell}{2L}}{\sin \frac{r-\ell}{2L}} \right|, \quad 0 \leq c < 2\pi L \quad (6)$$

for non-negative integers  $k \geq 0$ , while we obtain the real part of the self-energy as

$$\Re E(R = 2\pi Lk + c) = -32\pi^3 G_N D^2 L \begin{cases} f(k, c) & \text{if } 0 \leq c < \pi L \\ f(k+1, c-2\pi L) & \text{if } \pi L \leq c < 2\pi L \end{cases} \quad (7)$$

where we have defined the function  $f(k, c)$  by

$$f(k, c) \equiv k[3c^2 + 6\pi Lkc + (4k^2 - 1)\pi^2 L^2]. \quad (8)$$

Obviously  $\Im E(R)$  is periodic in  $R$  and vanishes at radii  $R = 2\pi Lk$  with  $c = 0$  (see Fig. 1 of Matsuda and Seki<sup>9</sup>). This implies that the spherical massive body becomes stable at the critical radii.

We also note that  $\Re E(R)$  identically vanishes for the very short range  $0 \leq R \leq \pi L$  and turns into negative values for the longer range  $R > \pi L$  (see Fig. 2 of Matsuda and Seki<sup>9</sup>). This suggests that for the above short range the ordinary gravitational potential due to the exchange of massless graviton is completely “screened” by the tachyonic graviton exchange of the infinite tower of the Kaluza-Klein mode.

### 2.4 Generic Features of Gravitational Stability and Screening Effect

So far we have chosen one particular type of mass density. One could construct an *onionlike hybrid model* for the mass density:

$$\rho_H = \begin{cases} \frac{D}{r} & \text{for } \max[0, (2km-1)\pi L] < r < (2km+1)\pi L \\ \frac{bD}{r} & \text{for } (2km+1)\pi L < r < (2(k+1)m-1)\pi L, \end{cases} \quad k = 0, 1, 2, \dots \quad (9)$$

where  $m$  is a fixed positive integer and  $b$  is a positive constant. One easily sees that, as  $b \rightarrow 1$  or when  $m = 1$ ,  $\rho_H(r) \rightarrow \rho(r) = D/r$ . By varying  $b$  and  $m$ , we obtain a variety of *onionlike hybrid models* with the common generic feature of gravitational stability which shows up in each model at critical radius  $R = 2\pi Lp$  with a corresponding positive integer  $p = km$ :

$$\Im E(R = 2p\pi L) = \Im E(R = 2km\pi L) = 0 \quad (10)$$

The gravitational screening effect turns out to be the generic feature of the Kaluza-Klein mode with an extra timelike dimension. This can be proved rigorously by the use of the summation formula:

$$i \sum_{n=1}^{\infty} \frac{1}{n} \left[ e^{i \frac{n}{L}(r+\ell)} - e^{i \frac{n}{L}(r-\ell)} \right] = \frac{1}{L} - i \ln \frac{\sin \frac{r+\ell}{2L}}{\sin \frac{r-\ell}{2L}}, \quad 0 < r - \ell \leq r + \ell < 2\pi L. \quad (11)$$

For  $0 < R \leq \pi L$  and any choice of  $\rho(r)$  we obtain from Eq. 3, Eq. 4 and Eq. 5:

$$E(R) = -i16\pi^2 G_N L \int_0^R dr \int_0^r d\ell \rho(r)\rho(\ell)r\ell \ln \frac{\sin \frac{r+\ell}{2L}}{\sin \frac{r-\ell}{2L}}, \quad 0 < R \leq \pi L \quad (12)$$

which proves that the real part of the self-energy vanishes identically for the short range  $0 < R \leq \pi L$  for any spherical mass density. We can therefore conclude generically that in the region  $0 < R \leq \pi L$  the gravitational force due to the ordinary massless graviton exchange is “screened” by the effect of the tachyonic graviton exchange of the Kaluza-Klein mode.

### 3 Conclusions

#### 3.1 Comments

We note that, in this paper, by “gravitational stability” we mean a stability of the self-energy state of a spherical massive body in pure Newtonian gravity, not the stability under metric perturbation usually referred to in Einstein gravity.

We also note that our results are classical, and it is not clear how the quantum loop corrections would modify the picture presented above. It is also not clear whether for the very short distance under our consideration a four-dimensional effective theory is well adapted to describe the physics for spherical massive bodies investigated here.

But with no concrete quantum gravity theory available for such small distances, we have performed our study by assuming that a conventional physical reasoning of our four-dimensional world is valid.

Though we have considered one extra timelike dimension, there is no reason why no additional timelike dimensions could exist. The extended case of  $D(\geq 1)$  extra times has been studied in detail<sup>10</sup>.

#### 3.2 Summary

Extra timelike as well as spacelike dimensions have rich physics structure for solving the standing problems in current particle theory such as the hierarchy problem and the cosmological constant problem.

We have made rigorous analytic studies on extra timelike dimensions and have shown on an analytic ground that some spherical bodies can be gravitationally stable at critical radii  $R = 2\pi Lp$  for some positive integer  $p$ .

We have also proved the generic property of massive bodies that for the range  $0 < R \leq \pi L$  the gravitational force due to the ordinary massless graviton exchange is screened by the Kaluza-Klein mode exchange of tachyonic gravitons.

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